[2] $uu' + 4yu^{\frac{5}{2}} = 0$ $\frac{1}{2}$ $\quad u = \frac{dy}{dx}, \quad u' = \frac{du}{dy}$

$$u\frac{du}{dy} = -4yu^{\frac{5}{2}}$$

$$\int u^{-\frac{3}{2}}\,du = \int -4y\,dy \quad \frac{1}{2}$$

$$-2u^{-\frac{1}{2}} = -2y^2 + C \quad \frac{1}{2}$$

$$\frac{dy}{dx} = u = (y^2+C)^{-2} \quad \frac{1}{2}$$

$$\int (y^2+C)^2\,dy = \int dx$$

$$\int (y^4 + 2Cy^2 + C^2)\,dy = x \quad \frac{1}{2}$$

$$\frac{1}{5}y^5 + \frac{2}{3}Cy^3 + C^2y + K = x \quad \frac{1}{2}$$

IS $u=0$ A SOL'N? BONUS $\frac{1}{2}$

$\frac{dy}{dx} = 0 \rightarrow y = C$

$y' = 0 \quad 0 + 4C(0)^{\frac{5}{2}} = 0$ ✓

$y'' = 0$

$y = C$ IS A SINGULAR SOLUTION (NOT IN THE FAMILY)

[3] $r^2+1=0 \to r=\pm i \to y_1=\cos(\ln x),\ y_2=\sin(\ln x)$ $\frac{1}{2}$

$\to W[y_1, y_2] = x^{-1}$ FROM QUIZ 3 [5][d]

$\frac{1}{2}$

$g = x^{-2}\sec^3(\ln x)$

$$y_p = -\cos(\ln x)\int \frac{x^{-2}\sec^3(\ln x)\sin(\ln x)}{x^{-1}}\,dx$$

$$+\sin(\ln x)\int \frac{x^{-2}\sec^3(\ln x)\cos(\ln x)}{x^{-1}}\,dx$$

MUST HAVE CORRECT ANTIDERIVATIVE BELOW ALSO $\frac{1}{2}$

$\frac{1}{2}$

$$= -\cos(\ln x)\int x^{-1}\sec^2(\ln x)\tan(\ln x)\,dx$$

$u=\tan(\ln x)$
$du = x^{-1}\sec^2(\ln x)\,dx$
$\int u\,du$

$$+\sin(\ln x)\int x^{-1}\sec^2(\ln x)\,dx$$

$\frac{1}{2}$

$v=\ln x$
$\int \sec^2 v\,dv$

MUST HAVE CORRECT ANTIDERIVATIVE ALSO

$\frac{1}{2}$

$$= -\cos(\ln x)\cdot\frac{1}{2}\tan^2(\ln x) + \sin(\ln x)\tan(\ln x)$$

$\frac{1}{2}$

$$= -\frac{1}{2}\frac{\sin^2(\ln x)}{\cos(\ln x)} + \frac{\sin^2(\ln x)}{\cos(\ln x)}$$

$$= \frac{1}{2}\sin(\ln x)\tan(\ln x)$$

$$y = \frac{1}{2}\sin(\ln x)\tan(\ln x) + c_1\cos(\ln x) + c_2\sin(\ln x)$$

$\frac{1}{2}$ $\frac{1}{2}$

[4] $r^7 + 8r^5 + 16r^3 = 0 \longrightarrow \boxed{r^3(r^2+4)^2 = 0 \longrightarrow r = 0, 0, 0, \pm 2i, \pm 2i}$ $\frac{1}{2}$

$$\boxed{y = c_1 + c_2 t + c_3 t^2 + c_4 \cos 2t + c_5 \sin 2t + c_6 t\cos 2t + c_7 t\sin 2t} \ \frac{1}{2}$$

$$+ \boxed{(At^3 + Bt^2 + Ct + E)t^3} + \boxed{Fe^{-2t}} +$$

$\frac{1}{2}$

$$\boxed{((Gt+H)\cos 2t + (Jt+K)\sin 2t)t^2}$$

$\frac{1}{2}$

0 POINTS, BUT SUBTRACT $\frac{1}{2}$ POINT IF INCORRECT OR MISSING

[5] $\frac{1}{2}$
$$\boxed{\begin{aligned}(3D+1)[x] \quad\quad\quad + D[y] &= t \quad ① \\ (2D+2)[x] + (D+1)[y] &= e^{2t} \quad ②\end{aligned}}$$

$$(D+1)[①]: \quad (D+1)(3D+1)[x] + D(D+1)[y] = (D+1)[t]$$
$$-D[②]: \quad -D(2D+2)[x] - D(D+1)[y] = -D[e^{2t}]$$

$$\boxed{(D^2+2D+1)[x] = x'' + 2x' + x = 1 + t - 2e^{2t}} \quad 1$$

$$\boxed{r^2 + 2r + 1 = 0 \longrightarrow r = -1, -1}$$

$$\boxed{x_h = c_1 e^{-t} + c_2 te^{-t}} \quad \frac{1}{2}$$

$$\boxed{x_p = At + B + Ce^{2t}} \quad \frac{1}{2}$$

$$x_p' = A + 2Ce^{2t}$$

$$\begin{aligned} x_p'' &= 4Ce^{2t} \\ +2x_p' &\quad +2A + 4Ce^{2t} \quad 1 \\ +x_p &\quad +At + B + Ce^{2t} \end{aligned}$$

$$= \boxed{At + (2A+B) + 9Ce^{2t}} \quad \frac{1}{2}$$
$$= t + 1 - 2e^{2t}$$

$$\boxed{x = t - 1 - \frac{2}{9}e^{2t} + c_1 e^{-t} + c_2 te^{-t}} \quad \frac{1}{2}$$

$\frac{1}{2}$
$$\boxed{\begin{aligned} &A = 1 \\ &2A + B = 1 \rightarrow B = 1 - 2A = -1 \\ &9C = -2 \rightarrow C = -\frac{2}{9}\end{aligned}}$$

$$-(2D+2)[①]: \quad -(2D+2)(3D+1)[x] - (2D+2)D[y] = -(2D+2)[t]$$
$$(3D+1)[②]: \quad (2D+2)(3D+1)[x] + (3D+1)(D+1)[y] = (3D+1)[e^{2t}]$$

$$\boxed{(D^2+2D+1)[y] = -2 - 2t + 7e^{2t}}$$

$$y_h = k_1 e^{-t} + k_2 te^{-t} \quad \frac{1}{2}$$
$$y_p = Et + F + Ge^{2t}$$

$$\boxed{y = -2t + 2 + \frac{7}{9}e^{2t} + k_1 e^{-t} + k_2 te^{-t}} \quad \frac{1}{2}$$

$\frac{1}{2}$
$$\boxed{\begin{aligned} &E = -2 \\ &2E + F = -2 \rightarrow F = -2 - 2E = 2 \\ &9G = 7 \rightarrow G = \frac{7}{9}\end{aligned}}$$

$$x' = 1 - \tfrac{4}{9}e^{2t} - c_1 e^{-t} - c_2 te^{-t} + c_2 e^{-t}$$

$$
\begin{aligned}
3x' &= 3 - \tfrac{12}{9}e^{2t} + (-3c_1 + 3c_2)e^{-t} - 3c_2 te^{-t} && (1)\\
+\ y' &\quad -2 + \tfrac{14}{9}e^{2t} - k_1 e^{-t} + k_2 e^{-t} - k_2 te^{-t} && (\tfrac{1}{2})\\
+\ x &\quad +t - 1 - \tfrac{2}{9}e^{2t} + c_1 e^{-t} + c_2 te^{-t} &&\\
\hline
&= t + (-2c_1 + 3c_2 - k_1 + k_2)e^{-t} + (-2c_2 - k_2)te^{-t} && (1)
\end{aligned}
$$

$$
(\tfrac{1}{2})\quad
\begin{aligned}
-2c_2 - k_2 &= 0 \longrightarrow k_2 = -2c_2\\
-2c_1 + 3c_2 - k_1 + k_2 &= 0 \longrightarrow k_1 = -2c_1 + 3c_2 + k_2\\
&\qquad\qquad\quad = -2c_1 + c_2
\end{aligned}
$$

$$
\begin{aligned}
x &= t - 1 - \tfrac{2}{9}e^{2t} + c_1 e^{-t} + c_2 te^{-t}\\
y &= -2t + 2 + \tfrac{7}{9}e^{2t} + (-2c_1 + c_2)e^{-t} - 2c_2 te^{-t}
\end{aligned}
\quad (1)
$$

OR

$$
\begin{aligned}
x &= t - 1 - \tfrac{2}{9}e^{2t} + \left(-\tfrac{1}{2}k_1 - \tfrac{1}{4}k_2\right)e^{-t} - \tfrac{1}{2}k_2 te^{-t}\\
y &= -2t + 2 + \tfrac{7}{9}e^{2t} + k_1 e^{-t} + k_2 te^{-t}
\end{aligned}
$$